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for a finite planet since 1973 !

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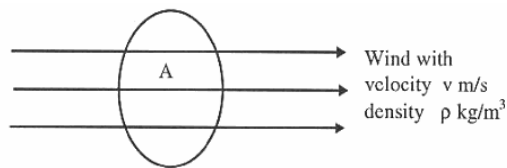
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## Wind Resources vs System Performance

### Energy in the Wind

The kinetic energy contained in wind may be calculated based on first principles as follows.



For a flow of air with velocity  $v$  and density  $\rho$  through a unit area  $A$  perpendicular to the wind direction, The kinetic energy per unit time is given by  $P$ :

$$P = \frac{1}{2} mv^2 = \frac{1}{2} (Av\rho) v^2 = \frac{1}{2} A\rho v^3$$

The variables  $\rho$  and  $v$  may not be constant but varies as follows:

- The air density  $\rho$  varies with altitude, for heights below approximately 6000m as

$$\rho = \rho_0 \exp(-0.297h/3048)$$

where  $\rho_0 = 1.225 \text{ kg/m}^3$  and  $h$  is in meters.

- The air density varies with the atmospheric pressure and temperature as

$$\rho = 3.4843 P/(T + 273)$$

where  $P$  is the atmospheric pressure (in kPa) and  $T$  is the ambient temperature (in °C).

- The air temperature defined for the standard atmosphere is a linearly decreasing function of altitude given by [1]:

$$T = 15 - 1.983(h/304.8)^\circ \text{ C}$$

### Types of WECS

Wind powered machines are usually classified according to their axis of rotation relative to the direction of the wind. The major categories of wind machines are:

- Horizontal axis and
- Vertical axis machines.

### Horizontal axis machines

Horizontal axis wind generators may be lift or drag systems. Drag systems (fig 1) have flat blades and rely on the drag force exerted by the wind on the surface of the blades. The blade tip speeds are less than the wind speed and generally are not very efficient. Lift systems are preferred as they have air-foil cross sections and utilize the lift force caused by the flow of air over the curved surfaces of their blades. This results in much

higher tip-to-wind speeds, enabling higher power-to-weight ratios which in turn favors the economics of such a system with a lower cost-to-power output ratio. Lift systems may have up to several blades as shown in fig 2.

**Advantages of horizontal axis systems:**

- high efficiency
- ability to furl by turning the rotor (blades) parallel to the wind direction
- low cut-in wind speeds
- generally lower cost-to-power output ratio

**Disadvantages of horizontal axis systems:**

- tail or yaw drive may be required; added complexity
- restricted servicing of generator and gearbox.

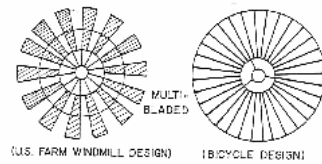


Fig 1 : Drag type propellers - multiblade

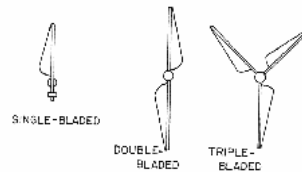


Fig 2 : Lift type propellers - single, double and Triple blades

**Vertical Axis Machines**

The first windmills were of the vertical axis type; however, they were never incorporated in any large scale operations like the horizontal axis machines. Typical vertical axis machines include the Darrius rotor (fig 3) and the Savonius rotor (fig 4).

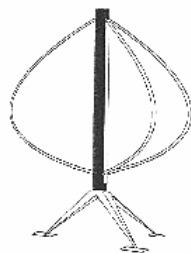


Fig 3 : Darrius rotor



Fig 4 : Savonius rotor

**Advantages of vertical axis systems:**

- Need for yaw control eliminated; reduced design complexity
- Ease of maintenance for ground mounted generator and gearbox
- Simple tower support design, structure and cost
- Simple blade design, and fabrication costs
- No pitch control for synchronous applications.

**Disadvantages of vertical axis systems:**

- Lower efficiency since blades must move through a "dead zone" where they do not generate torque
- Difficulty in controlling blade overspeed
- Not self starting, requiring additional mechanisms (such as a motor) to drive rotor up to speed when a predetermined wind speed is reached.

## Practical Output from WECS

- The Power output from a wind turbine may be expressed as

$$P = C_p \frac{1}{2} A \rho v^3$$

where

$C_p$  is the Power Coefficient of the turbine, defined as Power Output from turbine / Energy available in the wind.

$A$  is the swept rotor area

$v$  is the velocity of the wind at hub height, and varies with height as

where  $v_1$  and  $v_2$  are the wind speed altitudes at heights  $h_1$  and  $h_2$  respectively and  $a$  is the surface roughness factor.

- It is important to note that the wind power output is a cube function of the wind speed.

## Turbine Power Curve

The turbine power curve is a plot of the output of the turbine against the given wind speed. A typical curve is shown in figure 5.

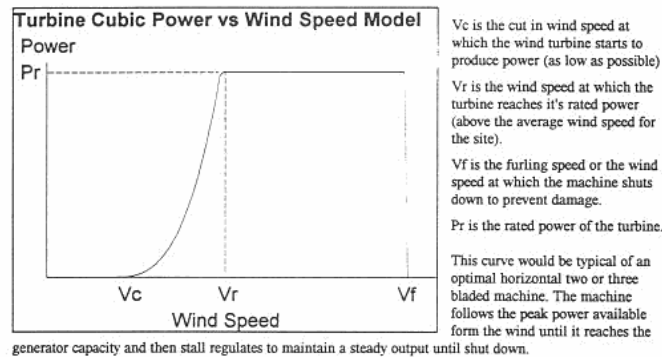


Figure 5 : Turbine Power Curve

A Turbine power output power is generally lower than that calculated from the power equation  $P = C_p \frac{1}{2} A \rho v^3$ . A minimum wind speed is necessary for starting to overcome friction. This minimum is the cut-in wind speed,  $V_c$ . At high wind speeds the turbine limits the output to prevent structural damage. The maximum wind speed at which this occurs is the cut-out or furling wind speed  $V_f$ .

A typical power output curve of an wind generator is shown in fig 6 [2]:

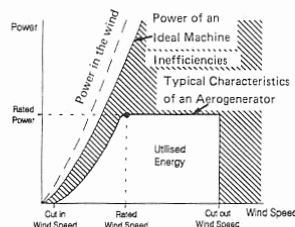


Fig 6 : Schematic power output curve versus wind speed for an aerogenerator

# Calculation of Wind Turbine Output

The power vs wind speed curve supplied by most wind turbine manufacturers is considered to be an instantaneous power point and as a result cannot be used to get the average output power using an average wind speed.

	Wind Speed	Wind Speed Cubed
	4	64
	5	125
	6	216
	7	343
Average	5.5	187
Average cubed	166	Error 11%

The mathematical technique of convolution exists to produce the correct value given the probability distribution function (Weibull) and the power curve. It is however difficult to perform on the curves we use and simpler methods have been developed to produce reasonable results.

## Using the Wind Speed Frequency Distribution and the Power Curve

Wind Speed	Power	Hours	Energy
0	0	472	0
1	0	298	0
2	12	147	1764
3	32	238	7616
4	249	293	72957
5	420	367	154140
6	576	500	288000
7	716	385	275660

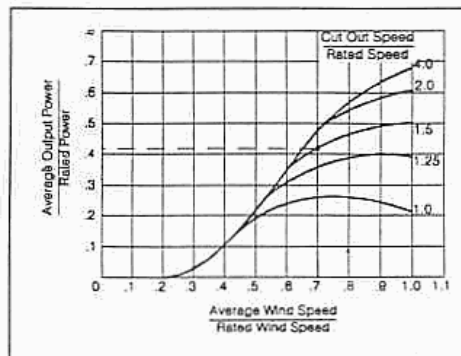
Portion of Spread Sheet

If wind speed distribution data is known and the power curve known then it is possible to calculate an approximate energy output for the turbine over a given period. This technique is easily applied to a spread sheet on a computer to cut down the time involved to perform the calculations.

The results are reasonably accurate. The smaller the bin size for the distribution the more accurate the result.

The difficulty with this method is getting hold of the wind distribution for a site with hourly resolution. If the data is not available then other techniques must be used.

Several techniques use idealise turbine power curves and frequency distribution curves so as to simplify the mathematics. The result of this is a family of curves shown below



All that is now required is the furling speed (cut out speed), the rated wind speed and the rated power.

Example; The annual average wind speed is 7m/s. The machine to be examined has a rated output of 5kW at 10m/s. The machine furls at 15m/s. Estimate the annual output of the machine.

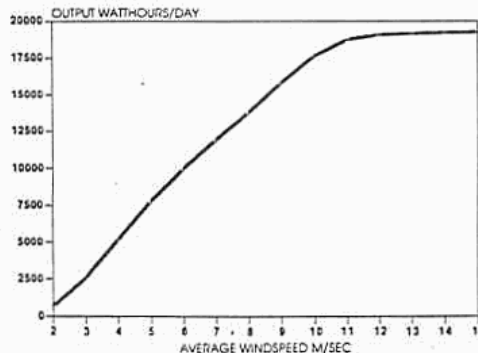
The ratio of cut out speed to rated speed is  $15/10 = 1.5$

The ratio of annual wind speed to rated wind speed is  $7/10 = 0.7$

Thus from the graph it can be seen that the output will be approximately 0.42 x rated output.

That is average output power will be

$0.42 \times 5\text{kW} = 2.10\text{kW}$ . The annual energy production would then be 18400 kWh



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